Linear and Quadratic Knapsack Optimization Problems

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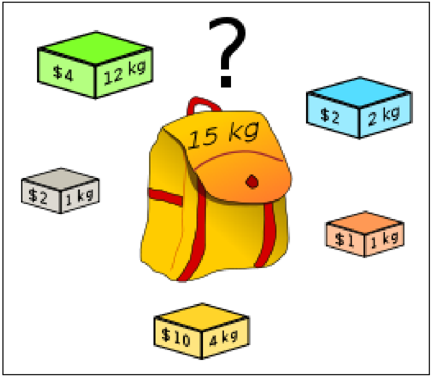

Figure: The knapsack problem

$$\max \quad \sum_{1=1}^{n} v_i x_i \tag{1}$$

$$\text{subject to} \quad \sum_{i=1}^{n} w_i x_i \leq W, x_i \in \{0, 1\} \tag{2}$$

$$\max_i w_i \leq W < \sum_{i=1}^{n} w_i$$

Choice of items to make where there are no items that depend on one another.

1. Sort items non-increasingly, according to v_i/w_i , i.e.

$$\frac{v_1}{w_1} \geq \frac{v_1}{w_1} \geq \cdots \frac{v_n}{w_n}$$

2. Fill items into the knapsack in the order $1, 2, \cdots, n$ until no items can be added.

1. Consider all possible sets of up to at most k items

$$\mathcal{F} = \{F \subset \{1, 2, \cdots, n\} : |F| \leq k, w(F) \leq W\}$$

2. For all $F \in \mathcal{F}$
   - Pack F to the knapsack
   - Greedily fill the remaining capacity
   - End
3. Return the most valuable set

$$\max \quad \sum_{j=1}^{n} c_j x_j \tag{3}$$

$$\text{subject to} \quad \sum_{j=1}^{n} a_{ij} x_j \leq b_i \; i \in \{1, 2, \cdots, m\}, x_j \in \{0, 1\} \tag{4}$$

Choice of projects to make where there are no projects that depend on one another.

$$\begin{cases} \max_x & \sum_{j=1}^{n} c_j x_j + \sum_{k=1}^{n-1} \sum_{j=k+1}^{n} d_{kj} x_k x_j, \quad c_j = d_{jj} \\ s.t. & \sum_{j=1}^{n} a_{ij} x_j \leq b_i, i = 1, \cdots, m, \\ & x \in \{0,1\}^n, \end{cases}$$

Choice of items to make where there are the relations between pair of items.

$$\begin{cases} \max_x & \sum_{i=1}^n \sum_{j=1}^n p_{ij} x_i x_j, \quad d_{ij} = p_{ij} + p_{ji} \\ s.t. & \sum_{j=1}^n w_j x_j \le W, i = 1, \cdots, m, \\ & x \in \{0,1\}^n, \qquad \max_j w_j \le W < \sum_{j=1}^n w_j \end{cases}$$

Choice of items to make where there are the relations between pair of items.